

Isotopic Yield in Cold Binary Fission of Even-Even $^{246-250}\text{U}$ Isotopes

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Abstract: Within the frame work of Coulomb and proximity potential model (CPPM), the cold binary fission of even-even uranium isotopes with the mass numbers $A=246, 248$ and 250 has been studied. With respect to the mass and charge asymmetries, high Q value and a minimum in the driving potential corresponds to the most favorable fragmentation in the binary fission process. A nucleus with doubly closed shell or near doubly closed shell is always appearing as the heaviest nucleus in the favored channel of the binary fission of all the mentioned isotopes. Hence the role of the nuclear shell structure in the formation of fission products is revealed through our study. It is found that highest yield are for the fragments with isotope of Sn ($Z=50$) as one fragment. The calculated half lives using Proximity 2000 are compared with the values obtained using Proximity 1977 version, and found that the values using Proximity 2000 for binary fission of $^{246-250}\text{U}$ even- A isotopes are one order greater than the values obtained using Proximity 1977.

Keywords - Cold binary fission, proximity potential, shell effect, cold reaction valley, half life.

I. Introduction

More than seventy-five years of research on nuclear fission have clearly shown that, the low energy fission of heavy elements ($Z>90$) was one of the most complex phenomena of nuclear reactions. Most of the nuclear reactions take place through the binary fission process, a low energy fission, where the fissioning nucleus ends up in two fission fragments and the fragments were formed after the fission barrier has been overcome. In 1939 Hahn et al., [1] discovered that the uranium atom was fragmented into two parts, which are more or less equal in size. Bohr and Wheeler [2] developed a theory of fission based on the liquid drop model.

Experimental studies of cold fission started in the early 80's by Signarbieux et al., [3] and found that the relative yields of different fragmentation modes are governed by the available phase space of the system at scission, determined by the nuclear structure properties of the fragments. The cold spontaneous fission of many actinide nuclei into fragments with masses from 70 to 160 were observed and studied [4-7] and found that in these cold decays both the final fragments were in the ground states and confirmed the theoretical predictions by Sandulescu et al., [8,9]. The first direct observation of cold fragmentation in the spontaneous fission of ^{252}Cf was carried out [6] using the multiple Ge-detector Compact Ball facility at Oak Ridge National Laboratory where four pairs of neutronless fragmentations that of $^{104}\text{Zr}-^{148}\text{Ce}$, $^{104}\text{Mo}-^{148}\text{Ba}$, $^{106}\text{Mo}-^{146}\text{Ba}$ and $^{108}\text{Mo}-^{144}\text{Ba}$ were observed.

The first investigation of the mass yield in cold fragmentation for $^{235}\text{U}(n_{\text{th}}, f)$, was carried out by Gibson et al., [10] by applying two semiconductor detectors to measure the energies of the two fission products in coincidence and observed a shift of the most probable light-fragment mass number from $A_L = 96$ to $A_L = 101$ with increasing light-fission-fragment kinetic energy. Fraser et al., [11] applied a double time-of-flight method for $^{235}\text{U}(n_{\text{th}}, f)$ and found the most probable mass split to be near $A_L/A_H = 102/134$, at light-fission-product kinetic energy of $E_L = 107$ MeV.

In this manuscript, our work aims to study the isotopic yield in binary fission of even-even $^{246-250}\text{U}$ isotopes by taking the interacting barrier as the sum of Coulomb and proximity potential. The calculated half lives using Proximity 2000 are compared with the values obtained using Proximity 1977 version, and found that the values using Proximity 2000 for binary fission of even-even $^{246-250}\text{U}$ isotopes are one order greater than the values obtained using Proximity 1977.

II. The Model

If Q value of the reaction is positive, the binary fission is energetically possible:

$$Q = M - \sum_{i=1}^2 m_i > 0 \quad (1)$$

Here M is the mass excess of the parent, m_i is the mass excess of the fragments. A parent nucleus exhibiting binary fission has the interacting potential, V given by:

$$V = \frac{Z_1 Z_2 e^2}{r} + V_p(z) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} \quad \text{for } z > 0 \quad (2)$$

Here Z_1 and Z_2 are the atomic numbers of the fission fragments, ' z ' is the distance between the near surfaces of the two fragments, ' r ' is the distance between these fragment centers and is given as $r = z + C_1 + C_2$, where, C_1 and C_2 are the Süsmann central radii of fragments. The term ℓ represents the angular momentum, μ the reduced mass and V_p is the proximity potential. The proximity potential V_p is given by Blocki et al., [12,13] as:

$$V_p(z) = 4\pi\gamma b \left[\frac{C_1 C_2}{(C_1 + C_2)} \right] \Phi\left(\frac{z}{b}\right) \quad (3)$$

where γ is the nuclear surface tension coefficient.

2.1 Proximity Potential 1977

The nuclear surface tension coefficient γ is given by:

$$\gamma = 0.9517[1 - 1.7826(N - Z)^2 / A^2] \text{ MeV/fm}^2 \quad (4)$$

Where N , Z and A represent neutron, proton and mass number of parent respectively, Φ represents the universal proximity potential [13] that could be given as:

$$\Phi(\xi) = -4.41e^{-\xi/0.7176}, \quad \text{for } \xi > 1.9475 \quad (5)$$

$$\Phi(\xi) = -1.7817 + 0.9270\xi + 0.0169\xi^2 - 0.05148\xi^3, \quad \text{for } 0 \leq \xi \leq 1.9475 \quad (6)$$

with $\xi = z/b$, where the width (diffuseness) of the nuclear surface $b \approx 1$ fm and Süsmann central radii C_i of fragments related to sharp radii R_i as:

$$C_i = R_i - \left(\frac{b^2}{R_i} \right) \quad (7)$$

For R_i we use semi empirical formula in terms of mass number A_i as [12]:

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \quad (8)$$

2.2 Proximity potential 2000

Myers and Swiatecki [14] modified Eq. (3) by using up-to-date knowledge of nuclear radii and surface tension coefficients using their droplet model concept. The important aim behind this effort was to eliminate disagreement in the case of barrier height between the results of Proximity 1977 and experimental data [14]. Using the droplet model [15], matter radius C_i was calculated as:

$$C_i = c_i + \frac{N_i}{A_i} t_i \quad (i=1, 2) \quad (9)$$

Where c_i denotes the half-density radii of the charge distribution and t_i is the neutron skin of the nucleus. The nuclear charge radius (denoted as R_{00} in Ref. [16]), is given by the relation:

$$\begin{aligned} R_{00i} &= \sqrt{5/3} \langle r^2 \rangle^{1/2} \\ &= 1.240A_i^{1/3} \left\{ 1 + \frac{1.646}{A_i} - 0.191 \left(\frac{A_i - 2Z_i}{A_i} \right) \right\} \text{ fm} \quad (i=1, 2) \end{aligned} \quad (10)$$

where $\langle r^2 \rangle$ represents the mean-square nuclear charge radius. According to Ref. [16], Eq. (10) was valid for the even-even nuclei with $8 \leq Z < 38$ only. For nuclei with $Z \geq 38$, the above equation was modified by Pomorski *et al.* [16] as:

$$R_{00i} = 1.256A_i^{1/3} \left\{ 1 - 0.202 \left(\frac{A_i - 2Z_i}{A_i} \right) \right\} \text{ fm} \quad (i=1, 2) \quad (11)$$

These expressions give good estimate of the measured mean square nuclear charge radius $\langle r^2 \rangle$. The half-density radius, c_i , was obtained from the relation:

$$c_i = R_{00i} \left(1 - \frac{7}{2} \frac{b^2}{R_{00i}^2} - \frac{49}{8} \frac{b^4}{R_{00i}^4} + \dots \right) \quad (i=1, 2) \quad (12)$$

Using the droplet model [15], neutron skin t_i reads as:

$$t_i = \frac{3}{2} r_0 \left(\frac{J I_i - \frac{1}{12} c_i Z_i A_i^{-1/3}}{Q + \frac{9}{4} J A_i^{-1/3}} \right) \quad (i=1, 2) \quad (13)$$

Here r_0 is 1.14 fm, the value of nuclear symmetric energy coefficient $J = 32.65$ MeV, and $c_1 = 3e_2/5r_0 = 0.757895$ MeV. The neutron skin stiffness coefficient Q was taken to be 35.4 MeV. The nuclear surface energy coefficient γ in terms of neutron skin was given as:

$$\gamma = \frac{1}{4\pi r_0^2} \left[18.63(\text{MeV}) - Q \frac{(t_1^2 + t_2^2)}{2r_0^2} \right] \quad (14)$$

where t_1 and t_2 were calculated using Eq. (13). The universal function $\Phi(\xi)$ was reported as:

$$\Phi(\xi) = \left\{ -0.1353 + \sum_{n=0}^5 \left[\frac{c_n}{n+1} \right] (2.5 - \xi)^{n+1} \right\} \quad \text{for } 0 < \xi \leq 2.5 \quad (15)$$

$$\Phi(\xi) = -0.09551 \exp[(2.75 - \xi)/0.7176] \quad \text{for } \xi \geq 2.5 \quad (16)$$

The values of different constants c_n were $c_0 = -0.1886$, $c_1 = -0.2628$, $c_2 = -0.15216$, $c_3 = -0.04562$, $c_4 = 0.069136$, and $c_5 = -0.011454$. For $\xi > 2.74$, the above exponential expression is the exact representation of the Thomas-Fermi extension of the proximity potential. This potential is termed as Proximity 2000.

The potential for the internal part (overlap region) of the barrier is given as:

$$V = a_0(L - L_0)^n, \text{ for } z < 0 \quad (17)$$

Here $L = z + 2C_1 + 2C_2$ and $L_0 = 2C$, the diameter of the parent nuclei. By the smooth matching of the two potentials at the touching point it is possible to determine the constants a_0 and n .

Using one-dimensional WKB approximation, the barrier penetrability P is given as:

$$P = \exp \left\{ -\frac{2}{\hbar} \int_0^b \sqrt{2\mu(V - Q)} dz \right\} \quad (18)$$

Instead of mass parameter, μ is used given as $\mu = mA_1A_2/A$, where ' m ' is the nucleon mass and A_1, A_2 are the mass numbers of binary fission fragments respectively. The turning point at $z=0$ represents touching configuration and outer turning point ' b ' is determined from the equation $V(b) = Q$. The half life time is given as:

$$T_{1/2} = \left(\frac{\ln 2}{\lambda} \right) = \left(\frac{\ln 2}{\nu P_0 P} \right) \quad (19)$$

where, $\nu = \left(\frac{\omega}{2\pi} \right) = \left(\frac{2E_v}{\hbar} \right)$ represent the number of assaults on the barrier per second and λ the decay constant. In

fission model the pre-formation factor, $P_0 = 1$ and E_v , the empirical vibration energy is given as [17];

$$E_v = Q \left\{ 0.056 + 0.039 \exp \left[\frac{(4 - A_2)}{2.5} \right] \right\} \quad \text{for } A_2 \geq 4 \quad (20)$$

The relative yield can be calculated as the ratio between the penetration probabilities of a given fragmentation over the sum of penetration probabilities of all possible fragmentation as follows:

$$Y(A_i, Z_i) = \frac{P(A_i, Z_i)}{\sum P(A_i, Z_i)} \quad (21)$$

III. Results, Discussion and Conclusion

Using the concept of cold reaction valley the binary fission of even-even $^{246-250}\text{U}$ isotopes has been studied. In the study, the structure of minima in the driving potential is considered. Most of the Q values are calculated using experimental mass excesses of Audi et al., [18] and some masses are taken from the table of KUTY [19]. The interaction potential is calculated as the sum of Coulomb and proximity potentials. Next the driving potential ($V-Q$) for a particular parent nuclei is calculated for all possible fission fragments as a function of mass and charge asymmetries respectively given as $\eta = \frac{A_1 - A_2}{A_1 + A_2}$ and $\eta_z = \frac{Z_1 - Z_2}{Z_1 + Z_2}$, at the touching configuration. For every fixed mass pair (A_1, A_2) a pair of charges is singled out for which the driving potential is minimized.

1.1 Cold reaction valley of even – even $^{246-250}\text{U}$ isotopes

Fig. 1 represents the plot for driving potential versus A_1 (mass of one fragment) for ^{246}U isotope. The occurrences of the mass-asymmetry valleys in this figure are due to the shell effects of one or both the fragments. The fragment combinations corresponding to the minima in the potential energy will be the most probable binary fission fragments.

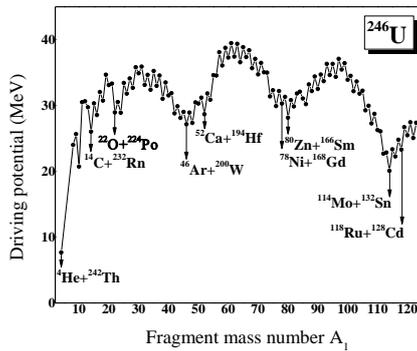


Fig. 1 Cold valley plot for ^{246}U isotope.

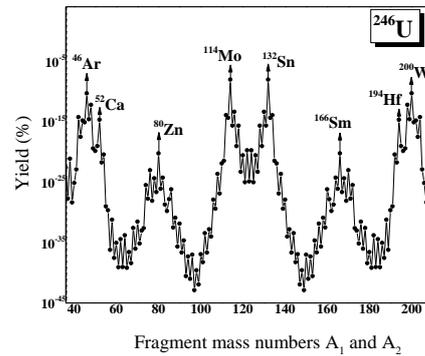


Fig. 2 The relative yield as a function of mass numbers A_1 and A_2 for ^{246}U isotope.

For ^{246}U in addition to the alpha particle ^{10}Be , $^{14,16}\text{C}$, $^{22,24}\text{O}$ are found to be the possible candidates for emission. Moving on to the fission region, there are three deep regions each consisting of few minima. For the first valley, the first minimum corresponds to the splitting $^{46}\text{Ar}+^{200}\text{W}$, while the other minima correspond to the splitting $^{48}\text{Ar}+^{198}\text{W}$ and $^{52}\text{Ca}+^{194}\text{Hf}$. From the cold valley approach the first minimum is due to the magic neutron shell $N = 126$ of ^{200}W and $N = 28$ of ^{46}Ar . In second valley the minima corresponds to the splitting $^{78}\text{Ni}+^{168}\text{Gd}$ and $^{80}\text{Zn}+^{166}\text{Sm}$ due to the presence of magic shell $Z=28$, $N = 50$ of ^{78}Ni and $N = 50$ of ^{80}Zn . A deep minima is found in the third valley for the splitting $^{114}\text{Mo}+^{132}\text{Sn}$ and their occurrence is attributed to the presence of doubly magic ^{132}Sn ($N = 82$, $Z = 50$).

Similarly cold valley is plotted for binary fission of ^{248}U and ^{250}U isotopes and the most probable fragment combinations are obtained in each case. It is clear that, as we move towards the symmetric fission region, we can see that the driving potential decreases with increase in mass number (ie. due to the increase in neutron number) of the parent nuclei. This is because in this region there is a chance for symmetric fission to occur (for e.g. $^{132}\text{Sn}+^{114}\text{Mo}$, $^{132}\text{Sn}+^{116}\text{Mo}$). This also stresses the role of double or near double magicity of the fragments.

1.2 Barrier penetrability and Yield calculation

The barrier penetrability for each charge minimized fragment combinations found in the cold valley are calculated using the formalism described above. Using eqn. (21) relative yield is calculated and is plotted as a function of fragment mass number A_1 and A_2 for all isotopes in Fig. 2-4. The most favorable fragment combinations for all the isotopes are obtained by calculating their relative yield.

For ^{246}U , ^{248}U and ^{250}U isotopes, $^{114}\text{Mo}+^{132}\text{Sn}$, $^{116}\text{Mo}+^{132}\text{Sn}$ and $^{118}\text{Mo}+^{132}\text{Sn}$ respectively are the most favored binary splitting. Other favored channels for ^{246}U are $^{46}\text{Ar}+^{200}\text{W}$ and $^{48}\text{Ar}+^{198}\text{W}$ fragment splitting whereas for ^{248}U are $^{48}\text{Ar}+^{200}\text{W}$ and $^{118}\text{Mo}+^{130}\text{Sn}$. In the case of ^{250}U the next higher yields are for the splitting $^{120}\text{Ru}+^{130}\text{Cd}$ and $^{52}\text{Ca}+^{198}\text{Hf}$.

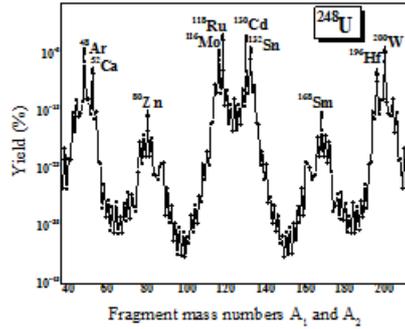


Fig. 3 The relative yield as a function of mass numbers A_1 and A_2 for ^{248}U isotope.

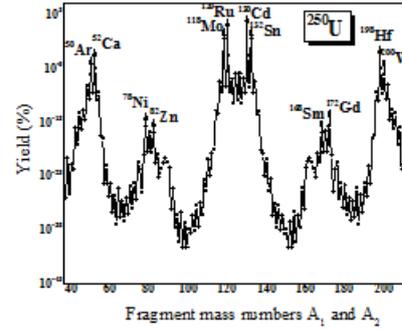


Fig. 4 The relative yield as a function of mass numbers A_1 and A_2 for ^{250}U isotope.

We have computed the half-lives for the binary fission of each nuclei using Proximity 2000 and is compared with that using Proximity 1977 shown in Fig. 5.

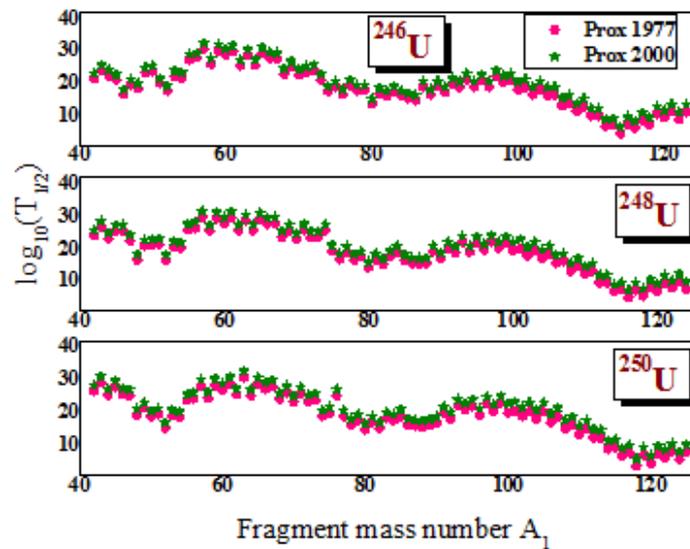


Fig. 5. A comparison of the computed $\log_{10}(T_{1/2})$ values vs. mass number A_1 using Proximity 2000 and Proximity 1977. $T_{1/2}$ is in seconds.

Our work reveals that, the presence of doubly magic or near doubly magic nuclei plays an important role in the binary fission of even-even $^{246-250}\text{U}$ isotopes. It is found that the magnitude of the relative yield increases with increase in mass number (i.e. due to the increase in neutron number) of the parent nuclei.

IV. Conclusion

To study the binary fragmentation of $^{246-250}\text{U}$ even-A isotopes, Coulomb and proximity potential is taken as the interacting barrier. In each case, the fragmentation potential and Q-values are calculated for all possible fission components. The relative yield can be calculated and hence the predicted favorable fragment combinations for the binary fission of all isotopes are discussed in detail. The role of the nuclear shell structure in the formation of fission products is revealed through our study.

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